

# Comparison Of Nonlinear Companding Transform (NCT) Algorithm With Adaptive-Active Constellation Algorithm PAPR Reduction Technique Of OFDM Signal

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## Abstract:

**Orthogonal Frequency Division Multiplexing (OFDM) is a method of multicarrier communication system in which a signal is split into several narrowband channels at different frequencies. . The OFDM signal has a noise like amplitude with a very large dynamic range; therefore it requires RF power amplifiers with a high Peak-to-Average Power Ratio (PAPR) or Peak-to-Average Ratio (PAR) [1]. The high PAPR increases the complexity of Analog-to-Digital (A/D) and Digital-to-Analog (D/A) converters and also lowers the efficiency of power amplifiers [1]. The Adaptive Active Constellation Extension (Adaptive ACE) Algorithm and Nonlinear companding scheme can be efficiently used to reduce the high PAPR for different modulation formats and subcarrier sizes without any complexity increase and bandwidth expansion. Therefore, this paper put forward comparison of two Scrambling PAPR reduction techniques namely Nonlinear Companding Transform and Adaptive-ACE Algorithm. The simulation results show that, Nonlinear Companding Transform gives better result for PAPR Reduction and improve the performance of OFDM systems including bit-error-rate and bandwidth.**

**Keywords** - Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Peak-to-Average Ratio (PAR), Adaptive Active Constellation Extension (Adaptive ACE), Nonlinear Companding Transform (NCT), SNR (Signal-to-Noise Ratio), CCDF (Complimentary Cumulative Distribution Function)

## 1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has many well known advantages including high data

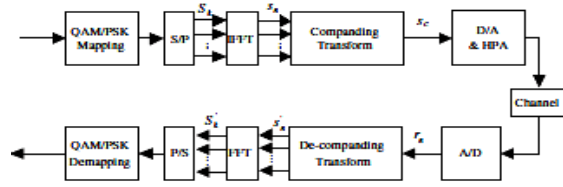
rates, high spectral efficiency, multipath delay spread tolerance, power efficiency and a very strong immunity to the frequency selective fading channels [1], [2]. Because of these advantages, OFDM widely deployed in many wireless communication standards such as Digital Video Broadcasting (DVB) and worldwide interoperability for microwave access (WiMAX) [3]. , One of the major problems is high Peak-to-Average Power Ratio (PAPR) of transmitted OFDM signals. This high PAPR forces the transmit power amplifier to have a large back-off in order to ensure linear amplification of the signal, which significantly reduces the efficiency of the amplifier. To overcome above mentioned serious Limitation, various solutions have been presented in the literature, which can be divided in two

categories—One category is to reduce the probability of generating high PAPR signals before doing multicarrier modulation, such as coding[3], Selective Mapping (SLM) [4], [5], and Partial Transform Sequence (PTS) [6], [7] known as distortion less techniques. And yet, most of these solutions have restrictions on system parameters such as the number of subcarriers, frame format, and constellation type. The other category is to deal with the signals after multicarrier modulation, such as clipping and filtering [8] and the companding transform[9]–[16], among which, the simplest and most widely used for reducing the PAPR of OFDM signals is clipping. Nevertheless, it causes additional clipping noise. In addition, this clipping noise becomes very significant with high modulation orders and seriously degrades the system performance [17], which makes Companding more suitable for high data rates applications. In this paper, a novel non-linear companding scheme is proposed. This scheme mainly focuses on compressing the large signals, while maintaining the average power constant by properly choosing the transform parameters. Finally, simulation results show that the proposed scheme offers a better PAPR reduction and BER performance than Adaptive Active Constellation Extension (Adaptive ACE) Algorithm .The remainder of the paper is organized as follows. Section II describes a typical OFDM system model and formulates the problem of high PAPR. Section III introduces the Adaptive- Active Constellation Extension PAPR Reduction Technique and its PAPR Vs CCDF and BER Vs SNR Simulation results. Section IV describes Nonlinear Companding Algorithm with its PAPR and BER simulation results. In Section V, the performance of the Nonlinear Companding scheme is compared with the Adaptive- Active Constellation Extension (Adaptive-ACE) through simulation. Finally, conclusions are given in Section VI.

## II OFDM SYSTEM AND PROBLEM FORMULATION.

Fig. 1 shows a generic OFDM system using the companding technique, the whole system bandwidth is

divided into many orthogonal sub-channels (with narrow bandwidth), and data symbols typically modulated by Phase Shift Keying (PSK or QAM are transmitted independently on the subcarriers.



**Fig. 1. Block diagram of an OFDM system with the companding transform.**

Let  $N$  denote the number of subcarriers used for parallel transmission and thus,  $S_k$  ( $0 \leq k \leq N - 1$ ) can be considered as the  $k$ th complex modulated symbol in a block of  $N$  symbols. The outputs  $S_n$  of the  $N$ -point inverse fast Fourier transform (IFFT) of  $S_k$  are the MCM signal samples over one symbol interval. For a real sequence output at the IFFT during one MCM symbol duration, the complex input to the IFFT has Hermitical symmetry and is considered as  $S_{N-k} = S_k^*$ , where  $k = 0, \dots, N/2 - 1$ . The results in the discrete time representation

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N/2-1} [a_k \cos(\frac{2\pi kn}{N}) - b_k \sin(\frac{2\pi kn}{N})] \quad (1)$$

Where  $n = 0, 1, \dots, N - 1$ . According to the central limit theorem, it follows that for large values of  $N$ ,  $S_n$  becomes Gaussian distribution with the probability density function (PDF)

$$f_{s_n}(s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{s^2}{2\sigma^2}\} \quad (2)$$

Where  $\sigma^2$  is the variance of the original MCM signals. Therefore, the signal  $S_n$  has distribution with the cumulative distribution function (CDF) as following

$$F_{s_n}(s) = \frac{1}{2} (1 + \operatorname{erf}(\frac{s}{\sqrt{2}\sigma}))$$

where  $\operatorname{erf}(x) = \int_0^x \frac{2}{\sqrt{\pi}} e^{-y^2} dy$ .

Then, the PAPR of MCM signals  $s_n$  in one symbol period is defined as

$$PAPR(s_n) = 10 \log_{10} \frac{\operatorname{Max}\{|s_n|^2\}}{\sigma^2} \quad (\text{dB}) \quad (4)$$

**A) PAPR and BER of Original OFDM Signal**

The Peak-to-Average Power Ratio (PAPR) of the original Orthogonal Frequency Division Multiplexing (OFDM) signals can be calculated by using the equation(5)

$$PAPR(x) \cong \frac{\max_{0 \leq n \leq N-1} |x_n|^2}{E[|x_n|^2]} \quad (5)$$

–Where PAPR - Peak-to-Average Power Ratio

The oversampled Orthogonal Frequency Division Multiplexing signal, denoted by  $x_n$ , is the Inverse Discrete Fourier Transform (IDFT) of the complex data symbols. The oversampled OFDM signal is obtained with the help of the modulation techniques like Quadrature Amplitude Modulation (QAM) or Phase Shift Keying (PSK) at the  $k$ th subcarrier. The oversampled OFDM signal is given by the equation .

$$X_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{kn}{N}}, \quad n = 0, 1, 2, \dots, N-1 \quad (5)$$

Where,  $x_n$  – Oversampled OFDM Signal  
 $N$  – Number of Subcarriers  
 $X_k$  – Complex Data Symbols using PSK or QAM at the  $k$ th Subcarrier

The Peak-to-Average Power Ratio (PAPR) of the Orthogonal Frequency Division Multiplexing (OFDM) systems in dB is given by the equation 6 [7], [29].

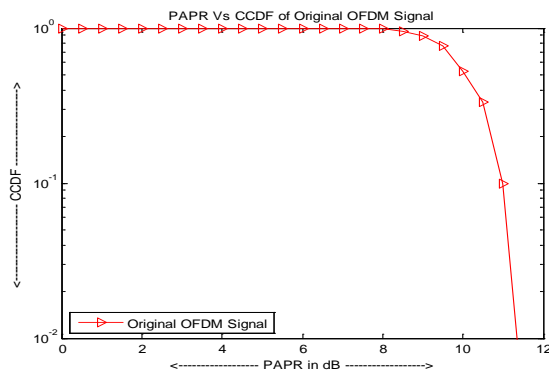
$$PAPR \text{ in dB} = 10 \log_{10}(\text{PAPR}) \quad (6)$$

Where, PAPR – Peak-to-Average Power Ratio

The Peak-to-Average Power Ratio (PAPR) of any OFDM signal is to be calculated in terms of the Complimentary Cumulative Distribution Function (CCDF) as the CCDF is one of the most frequently used performance measures for the PAPR reduction techniques

**Simulation Results of Original OFDM Signal**

1) PAPR Of Original OFDM Signal:



**Fig.1 PAPR VS CCDF Of OFDM Signal**

Fig.1 shows that Peak-to-Average Power Of OFDM signal is 11.8 dB at CCDF  $10^{-2}$  i.e. it is very high PAPR which increases complexity of high power amplifier (HPA) at transmitter side.

**B) Signal-to-Noise Ratio(SNR) of Original OFDM Signal.**

Signal-to-Noise Ratio (SNR) is defined as the power ratio between a signal i.e., a meaningful information or data and the background noise i.e., an unwanted signal. The SNR is given by the equation (7).

$$SNR = \frac{P_{\text{signal}}}{P_{\text{noise}}} \quad (7)$$

Where, SNR – Signal-to-Noise Ratio

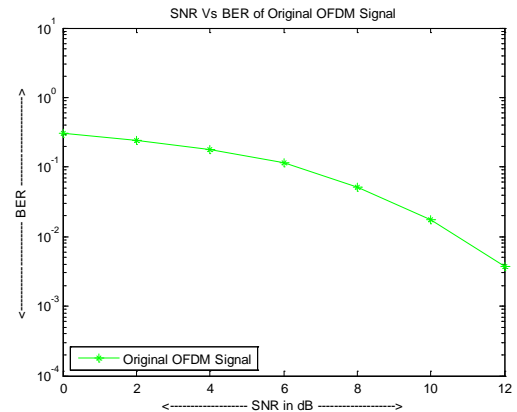
$P_{\text{signal}}$  – Signal Power

$P_{\text{noise}}$  – Noise Power

**C) Bit Error Rate (BER) Of Original OFDM Signal.**

The Bit Error Rate or Bit Error Ratio (BER) is defined as the number of bit errors divided by the total number of transferred bits during a studied time interval. BER is a unit less performance measure, often expressed as a percentage number.

2) BER Vs SNR of Original OFDM Signal



**Fig 2.BER Vs SNR of Original OFDM Signal**

Fig 2 show that BER is  $10^{-2}$  at SNR 12 dB i.e. it is very high at receiver side

**III Adaptive Active Constellation Extension Algorithm (Adaptive-ACE).**

The main objective of the Adaptive Active Constellation Extension (Adaptive ACE) algorithm for reducing the Peak-to-Average Power Ratio (PAPR) is to control both the clipping level and the convergence factor at each step and thereby minimize the peak power signal whichever is greater than the initial target clipping level [3].

The Adaptive Active Constellation Extension (Adaptive ACE) algorithm can be initialized by selecting the parameters namely the target clipping level, denoted by A and the number of iterations, denoted by i. In the first step, the iteration is taken as two i.e.,  $i = 2$  and the initial target clipping level is to be taken as A [3].

The predetermined clipping level, denoted by A, is related to the target clipping ratio,  $\gamma$  and given is by the equation 6 [3].

$$\gamma = \frac{A^2}{E\{|x_n|^2\}} \quad (8)$$

Where,  $\gamma$  – Target Clipping Ratio

A – Predetermined Clipping Level

$x_n$  – Oversampled OFDM signal

The clipping of the peak signal results to distortion of the original OFDM signal. The distortion of the original signal can be assumed as the noise, which results to an unreliable communication between the transmitter and the receiver. The distortion caused by clipping the original OFDM signal is categorized into two types, namely –

- In-Band Distortion.
- Out-of-Band Distortion.

The in-band distortion results in the system performance degradation and cannot be reduced, while, the out-of-band distortion can be minimized by filtering the clipped

signals. The signal obtained after filtering the clipped signal is given by the equation 9 [3].

$$X^{(i+1)} = \mu C^{(i)} + \tilde{C}^{(i)}$$

The anti-peak signal at the  $i^{th}$  iteration generated for the PAPR reduction, denoted by  $\tilde{C}^{(i)}$ , is given by the equation 9 [3].

$$\tilde{C}^{(i)} = T^{(i)} C^{(i)} \tag{9}$$

Where,  $\tilde{C}^{(i)}$  – Anti-Peak Signal at the  $i^{th}$  iteration  
 $T^{(i)}$  – Transfer Matrix at the  $i^{th}$  iteration  
 $C^{(i)}$  – Peak Signal above the Pre-Determined Level

The transfer matrix at the  $i^{th}$  iteration, denoted by  $T^{(i)}$ , used for generating the anti-peak signal is given by the equation 10 [3].

$$T^{(i)} = \hat{Q}^{*(i)} \hat{Q}^{(i)} \tag{10}$$

Where,  $T^{(i)}$  – Transfer Matrix at the  $i^{th}$  iteration  
 $\hat{Q}^{*(i)}$  – Conjugate of Constellation Order  
 $\hat{Q}^{(i)}$  – Constellation Order

The original Orthogonal Frequency Division Multiplexing (OFDM) signal, denoted by  $X_n$ , is to be clipped in order to reduce the peak signals. The clipping signal is given by the equation 11 [3].

$$c_n^{(i)} = \begin{cases} (|x_n^{(i)}| - A)e^{j\theta_n}, & |x_n^{(i)}| > A \\ 0, & |x_n^{(i)}| \leq A \end{cases} \tag{11}$$

Where,  $c_n^{(i)}$  – Clipping Sample  
 $A$  – Predetermined Clipping Level  
 $\theta_n = \arg(-x_n^{(i)})$

The clipping level, denoted by  $A$ , for the next iteration is given by the equation 12 [3].

$$A^{(i+1)} = \mu A^{(i)} + \nabla_A \tag{12}$$

Where,  $A^{(i+1)}$  – Next Iteration Level  
 $A^{(i)}$  – Present Iteration Level  
 $\mu$  – Convergence Factor  
 $\nabla_A$  – Gradient with respect to  $A$

The Peak-to-Average Power Ratio by the Adaptive Active Constellation Extension (Adaptive ACE) algorithm is to be calculated for the Orthogonal Frequency Division Multiplexing (OFDM) signal which is obtained after filtering the clipped signal .

### Simulation Results of Adaptive Active Constellation Extension Algorithm.

#### 1) PAPR Of Adaptive-ACE Algorithm

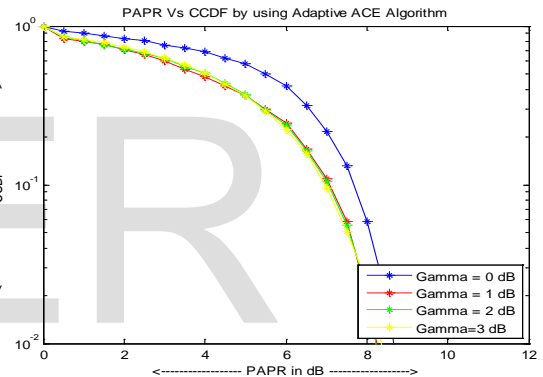
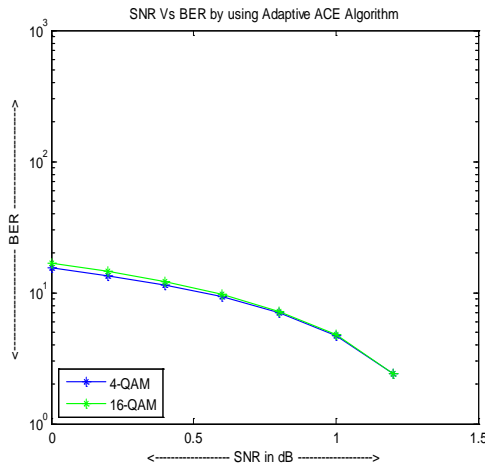


Fig 3.PAPR Vs CCDF Of Adaptive –ACE algorithm

Fig 3 shows that the Peak-to-Average Power Ratio (PAPR) of the Orthogonal Frequency Division Multiplexing (OFDM) signal obtained by using the Adaptive Active Constellation Extension (Adaptive ACE) algorithm is equal to 8.5 dB, 8.2dB, 8.2 for all the target clipping ratios i.e., for  $\gamma = 0$  dB or  $\gamma = 1$  dB or  $\gamma = 3$  dB with a Complimentary Cumulative Distribution Function (CCDF) of  $10^{-2}$  or 0.01.

#### 2) Bit Error Rate (BER) Of Adpative –ACE Algorithm.



**Fig.4 BER Vs SNR of Adaptive ACE Algorithm**

From the fig-4 shows that the Signal-to-Noise Ratio (SNR) of the Orthogonal Frequency Division Multiplexing (OFDM) signal obtained by the Adaptive Active Constellation Extension (Adaptive ACE) algorithm is equal to 1.2 dB at a Bit Error Rate (BER) of  $10^{-0.4}$  for different constellation orders like 4-Quadrature Amplitude Modulation (4-QAM) and 16-Quadrature Amplitude Modulation (16-QAM).

#### IV Nonlinear Companding Transform

##### Algorithm.

The Nonlinear Companding Transform Algorithm is also known as Exponential Companding Technique used for reducing the high Peak-to-Average Power Ratio (PAPR). The idea of companding comes, from the use of companding in Speech Processing. Since, the Orthogonal Frequency Division Multiplexing (OFDM) signal is similar to that of the speech signal, in the sense that large signals occur very infrequently, the same companding technique can be used to improve the OFDM transmission performance [29].

The key idea of the Exponential Companding Transform is to effectively reduce the Peak-to-Average Power Ratio (PAPR) of the transmitted or the companded Orthogonal Frequency Division Multiplexing (OFDM) signals by transforming the statistics of the amplitudes of these signals into uniform distribution. The uniform distribution of the signals can be obtained by compressing the peak signals and expanding the small signals. The process of companding enlarges the amplitudes of the small signals, while the peaks remain unchanged. Therefore, the average power is increased and thus the Peak-to-Average Power Ratio (PAPR) can be reduced [29].

The Nonlinear Companding Transform can also eliminate the Out-of-Band Interference (OBI), which is a type of distortion caused by clipping the original OFDM signals. The other advantage of the companding transform is that, it can maintain a constant average power level. The proposed scheme can reduce the PAPR for different modulation formats and sub-carrier sizes without increasing the system complexity and signal bandwidth. The Exponential Companding Transform also causes less spectrum side-lobes [29].

The companded signal obtained by using the Exponential or Nonlinear Companding Transform is given by the equation 13 [29].

$$h(x) = \text{sgn}(x) \alpha \sqrt{1 - \exp\left(-\frac{x^2}{\sigma^2}\right)} \quad (13)$$

Where,  $h(x)$  – Companded Signal obtained by Nonlinear Companding Transform

$\text{sgn}(x)$  – Sign Function

$\alpha$  – Average Power of Output Signals

$x$  – Original OFDM Signal

The average power of the output signals, denoted by  $\alpha$ , is required in order to maintain the average amplitude of both the input and output signals at the same level. The average power of the output signals is given by the equation 14 [29].

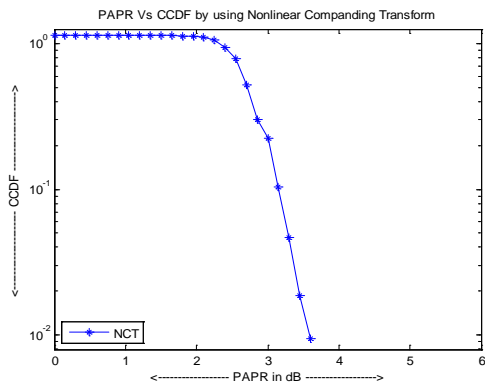
$$\alpha = \left( \frac{E[|s_n|^2]}{E \left[ \sqrt{1 - \exp\left(-\frac{|s_n|^2}{\sigma^2}\right)} \right]^2} \right)^{\frac{d}{2}} \quad (14)$$

Where,  $\alpha$  – Average Power of Output Signals

$d$  – Power of the amplitude of the Companded Signal

#### Simulation Results of Nonlinear Companding Algorithm .

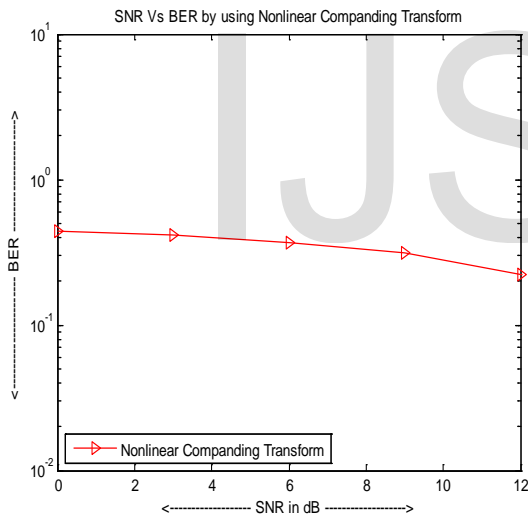
**1) PAPR & CCDF Nonlinear Companding Transform**



**Fig.5 PAPR Vs CCDF using NCT .**

From the fig.5 the Peak-to-Average Power Ratio (PAPR) of the Orthogonal Frequency Division Multiplexing (OFDM) signals obtained by using the Exponential Companding Transform is reduced to 3.5 dB with a (CCDF) of  $10^{-2}$  or 0.01.

**2) BER VS SNR using NCT.**



**Fig-6 BER Vs SNR using NCT Algorithm**

From the fig -6, the Signal-to-Noise Ratio (SNR) of the companded Orthogonal Frequency Division Multiplexing (OFDM) signals obtained by using the Exponential Companding Transform is equal to 12 dB at a Bit Error Rate (BER) of  $10^{-0.6}$  for 16-Quadrature Amplitude Modulation (16-QAM).

**Table 1 – Comparison of PAPR (in dB) at CCDF= $10^{-2}$  and BER for different techniques**

Sr.No	PAPR(dB) at CCDF = $10^{-2}$	BER
Original OFDM signal	10.8	$10^{-2}$ at SNR=12(dB)
Adaptive –Active Constellation Extension algorithm	8.5 at $\alpha=0$ dB	$10^{-0.1}$ at
	8.2 at $\alpha=1$ dB	SNR =1.3 (dB)
	8.0 at $\alpha=2$ dB	
Nonlinear Companding Transform Algorithm	3.5	$10^{-0.6}$ at SNR=12dB

The Peak-to-Average Power Ratio (PAPR) of the Orthogonal Frequency Division Multiplexing (OFDM) system is equal to 11.8 dB (approximately 12 dB) without using any algorithm i.e., by using the basic formula of PAPR for the original OFDM signal. The existing techniques for reducing the high Peak-to-Average Power Ratio (PAPR) of the OFDM systems are Adaptive Active Constellation Extension (Adaptive-ACE) Algorithm and Exponential Companding Transform. The Adaptive Active Constellation Extension (Adaptive ACE) algorithm reduced the Peak-to-Average Power Ratio (PAPR) to 8.5 dB for all the target clipping ratios, which says that the low target clipping ratio problem faced is eliminated completely.

The Nonlinear Companding Transform comp ands the original OFDM signal i.e., compresses the peak signals and expands the small signals. Then the PAPR basic formula is applied on the companded signal. The Peak-to-Average Power Ratio (PAPR) is reduced to 3.5 dB by using the Exponential Companding Transform.

From the table 1, the Signal-to-Noise Ratio (SNR) of the OFDM signal is reduced by using Nonlinear Companding Transform (NCT) PAPR reduction technique like Clipping-Based Active Constellation Extension (CB-ACE) Algorithm , but the SNR is drastically reduced by using the Adaptive Active Constellation Extension (Adaptive ACE) Algorithm.

From the table 1, the Bit-Error Rate (BER) of the OFDM signal is drastically increased by using the PAPR reduction technique like Adaptive Active Constellation Extension (Adaptive ACE) Algorithm but the BER is slightly increased by using the Nonlinear Companding Transform, when compared with the original OFM signal.

**V CONCLUSION**

The Adaptive Constellation Extension (Adaptive-ACE) Algorithm reduces the high Peak-to-Average Power

Ratio (PAPR) by clipping and filtering the original OFDM signal. The Adaptive-ACE Algorithm results to peak re-growth, Out-of-Band Interference (OBI), low clipping ratio problem, increase in the Bit Error Rate (BER) and decrease in the Signal-to-Noise Ratio (SNR). The Nonlinear Companding Transform improves the Bit Error Rate (BER) and minimizes the Out-of-Band Interference (OBI) in the process of reducing the Peak-to-Average Power Ratio (PAPR) effectively by compressing the peak signals and expanding the small signals.

Nonlinear commanding transforms can effectively reduce PAPR for different modulation formats and subcarrier sizes without any complexity increase and bandwidth expansion.

Hence, by reducing the Peak-to-Average Power Ratio (PAPR), the complexity of the Analog-to-Digital Converter (ADC) and Digital-to-Analog Converter (DAC) can be reduced. The reduced Peak-to-Average Power Ratio (PAPR) also increases the efficiency of the Power Amplifiers.

Finally, Simulation results have shown that the Nonlinear Companding Transform (NCT) algorithm could offer better performance in-terms of PAPR reduction, Power spectrum and without increase Complexity Of system performance than Adaptive- ACE PAPR Reduction Technique.

## VI ACKNOWLEDGMENT

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